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COMMENTS ON THE DOUBLY ASYMPTOTIC APPROXIMATION: I. KINETIC ENERGY AT PRESSURE CUTOFF

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This report uses the Doubly Asymptotic Approximation to consider the shock wave energy available for damage in structures attacked by underwater warheads. In particular, we estimate the cutoff kinetic energy for an infinite unrestrained air-backed flat plate on which an exponentially decaying acoustic wave is incident. A comparison is given for the titanium and steel at equal plate thickness. Similarly, a comparison is given for conventional warheads and copper-lined shaped charge warheads with equal peak pressures. Over such a range of conditions there is a greatly varying ratio of pressure decay times to

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FOREWORD

The ultimate objective of the work presented here is the direct prediction of the onset and details of submarine pressure hull rupture caused by underwater explosive attack. The immediate objective of this work is to bring out some of the implications of an important analytical tool known as the Doubly Asymptotic Approximation. The results reported here provide an indication of the effect of using light structural materials such as titanium and of using nonconventional warheads such as copper-lined shaped charge warheads.

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INTRODUCTION

The ultimate objective of the present work is the prediction of submarine structural response to underwater explosions. A major element in the physics is fluid-structure interaction by virtue of which pressures exerted by the fluid are relieved as the structure complies. Even though fluid-structure interaction comprises a formidable modeling problem, there has been some success treating it using a simplification known as the Doubly Asymptotic Approximation (DAA) [1]. Here we illustrate some consequences of the DAA in a model problem involving an exponentially decaying acoustic wave impinging on an infinite air-backed unrestrained flat plate, as illustrated in Figure 1.

DOUBLY ASYMPTOTIC APPROXIMATION

For the model problem the incident pressure is subject to exponential decay typical of underwater explosions:

$$p_I = p_0 \exp(-t/\theta)$$

The total pressure on the plate may be thought of as the sum of the incident pressure, the reflected pressure, and the radiated pressure arising from plate compliance:

$$p_T = p_I + p_R + p_r$$

The DAA uses the approximations

$$p_R = p_I$$
 $p_r = -\rho cv$

where ρ is the water density, c is the acoustic wave speed, and v is the plate velocity. $^{1\!-\!3}$

The equilibrium equation becomes

$$m\frac{dv}{dt} = p_T$$

$$= 2p_0 \exp(-t/\theta) - \rho cv$$

Geers, T. L., "Transient Response Analysis or Submerged Structures," in Finite Element Analysis of Transient Nonlinear Structural Behavior, ASME, 1975.

Kennard, E. H., "The Effect of Pressure Wave on a Plate or Diaphragm," in <u>Underwater Explosion Research</u>, Office of Naval Research, 1950.

Taylor, G. I., "The Distortion Under Pressure of a Diaphragm which is Clamped Along its Edge and Stressed Beyond its Elastic Limit," in <u>Underwater Explosion</u> Research, Office of Naval Research, 1950.

where m is the plate mass per unit area. Now using the dimensionless quantities

$$w = v/c$$
 $\tau = t/\theta$

$$\kappa = \rho c\theta/m$$
 $v = 2p_0\theta/cm$

the equation (1) may be rewritten

$$\frac{dw}{d\tau} + \kappa w \approx v \exp(-\tau). \tag{2}$$

With the initial condition w(o) = o, the solution of (2) is

$$w = \frac{\sigma}{1-\kappa} \left[\exp(-\kappa \tau) - \exp(-\tau) \right]$$
 (3)

KINETIC ENERGY AT CUTOFF

Cutoff occurs when the total pressure due to the shock loading vanishes. We assume that no shockwave energy is transferred to the plate after cutoff. However, in reality, a considerable amount of energy may be transferred by afterflow, bubble pulses, etc. This has to be the subject of a subsequent analysis. The kinetic energy at cutoff should be considered the shock wave energy available for damage. Pressure cutoff therefore occurs when

$$\rho cv = 2p_0 \exp(-t/\theta)$$

and hence when

$$\kappa w = v \exp(-\tau). \tag{4}$$

Substituting (4) into (3) and solving for the cutoff time τ_{c} leads to

$$\tau_c = -kn \kappa / (1-\kappa)$$

The cutoff velocity is $\mathbf{w}_{\mathbf{C}}$ where

$$w_{C} = \frac{v}{\kappa} \exp(-\tau_{C})$$

$$= v \left(\frac{1}{1-\kappa}\right)$$

Most importantly, the kinetic energy at cutoff is

$$K_{C} = \frac{1}{2} m v_{C}^{2}$$

$$= \mu \theta \kappa^{\left(\frac{1+\kappa}{1-\kappa}\right)}$$
(5)

where $\mu = 2p^2_0/\rho c$

The prominence of the quantity $\kappa = cc\theta/m$ illustrates the physics very succinctly. It is nothing but the ratio of pressure decay times to structural (inertial) response times.

COMPUTATIONAL RESULTS

Computations are now presented for two cases.

Case 1: Here the time constant θ is held fixed and only the mass m is varied. This corresponds to comparisons of the effect of using titanium in place of steel.

An appropriate dimensionless counterpart of the cutoff kinetic energy is given by $\chi_1 = K_c/\mu\theta$, for which

$$\chi_1 = \kappa^{\frac{1-\kappa}{1-\kappa}}$$

The limiting cases of χ_1 are

$$\lim_{\kappa \to 0} \chi_1 = 0$$

$$\lim_{\kappa \to \infty} \chi_1 = 0$$

$$\lim_{\kappa \to \infty} \chi_1 = e^{-2} = .135$$

The maximum value of x_1 corresponds to $\kappa=1$. It should be recalled that $\kappa=\rho c\theta/m$ and so κ is inversely proportional to m.

The dependence of kinetic energy at cutoff on plate mass, represented by the function $x_1(\kappa)$, is shown in Figure 2. Illustrative examples of these results will be given in the next section.

Case 2: Here the mass m is fixed, while only the time constant θ is allowed to vary. Now a dimensionless counterpart of the cutoff kinetic energy is $x_2 = K_C/\lambda$ where $\lambda = 2p^2_0 m/\rho^2 c^2$, for which

$$X_2 = \kappa^{2/1-\kappa}$$

This quantity has the limiting cases

$$\lim_{\kappa \to 0} \chi_2 = 0 \qquad \qquad \lim_{\kappa \to \infty} \chi_2 = 1$$

$$\lim_{\kappa \to 0} \chi_2 = e^{-2} = .135$$

Unlike $\chi_1^{}$, the maximum of $\chi_2^{}$ occurs at $\kappa\!\to\!\!\infty$ and not at κ =1.

The dependence of kinetic energy at cutoff on decay time θ , represented by the function $\chi_2(\kappa)$, is shown in Figure 3. Recall that κ is directly proportional to θ . Illustrative examples of these results will be given in the next section.

EXAMPLES

For the conventional UNDEX explosive represented by 50 pounds of TNT at 300 feet standoff, reference 2 gives the value

$$\theta_1 = 7 \cdot 10^{-4} \text{ sec}$$

For a copper-lined shaped charge warhead fired under water, we found some data from which we estimated the decay time as

$$\theta_2 = 3.5 \cdot 10^{-5} \text{ sec}$$

For a one inch thick steel plate

$$(m/\rho c)_1 = 1.3 \cdot 10^{-4} \text{ sec}$$

while for a one inch thick titanium plate

$$(m/\rho c)_2 = 0.78 \cdot 10^{-4} sec$$

We may form the following table:

	Conventional	Copper Jet	
	κ _• χ ₁ /.135, χ ₂	$\kappa_{,\chi_{1}}$ /.135, χ_{2}	
steel	5.38, 0.638, 0.464	0.27, 0.759, 0.028	
titanium	8.97, 0.476, 0.579	0.45, 0.903, 0.055	

The values of $\chi_1/.135$ suggest that a titanium flat plate develops about 15-30% less cutoff kinetic energy than a steel flat plate with the same plating thicknesses. The values of χ_2 suggest that, for both metals and a fixed incident pressure amplitude p_0 , the cutoff kinetic energy for a shaped charge jet is a small fraction of that for a conventional warhead.

DISCUSSION

The Doubly Asymptotic Approximation has been used to investigate the consequences of certain target and warhead variations on the shock wave energy available for damage. At similar plate thickness, a titanium structure may develop 15-30% less energy than a steel structure. At the same incident pressure amplitudes, a copper-lined shaped charge warhead may introduce only a small fraction of the energy introduced by a conventional warhead.

REFERENCES

- 1. Geers, T. L., "Transient Response Analysis of Submerged Structures," in Finite Element Analysis of Transient Nonlinear Structural Behavior, ASME, 1975.
- 2. Kennard, E. H., "The Effect of a Pressure Wave on a Plate or Diaphragm," in Underwater Explosion Research, Office of Naval Research, 1950.
- 3. Taylor, G. I., "The Distortion Under Pressure of a Diaphragm which is Clamped Along its Edge and Stressed Beyond its Elastic Limit," in <u>Underwater Explosion Research</u>, Office of Naval Research, 1950.

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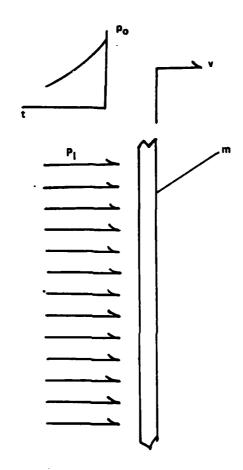
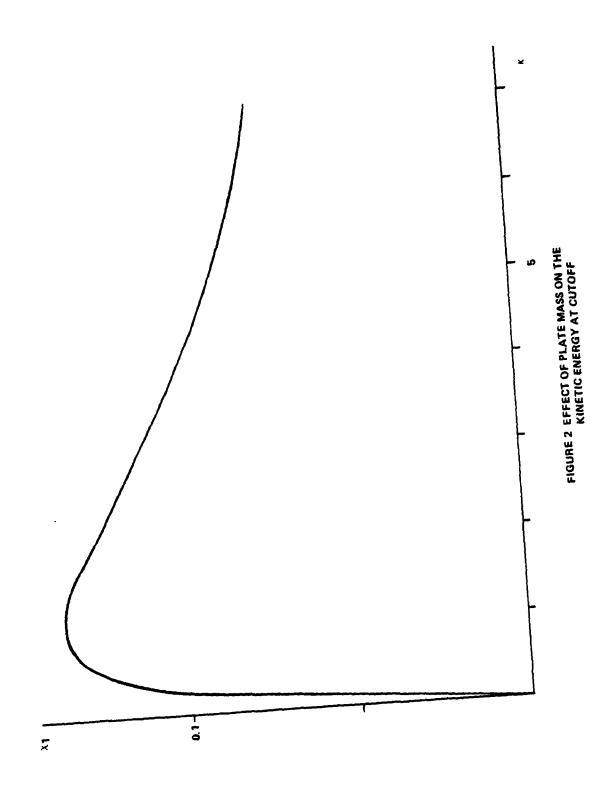


FIGURE 1 EXPONENTIALLY DECAYING PLANE WAVE INCIDENT ON AN UNRESTRAINED AIRBACKED PLATE IN AN ACOUSTIC FLUID



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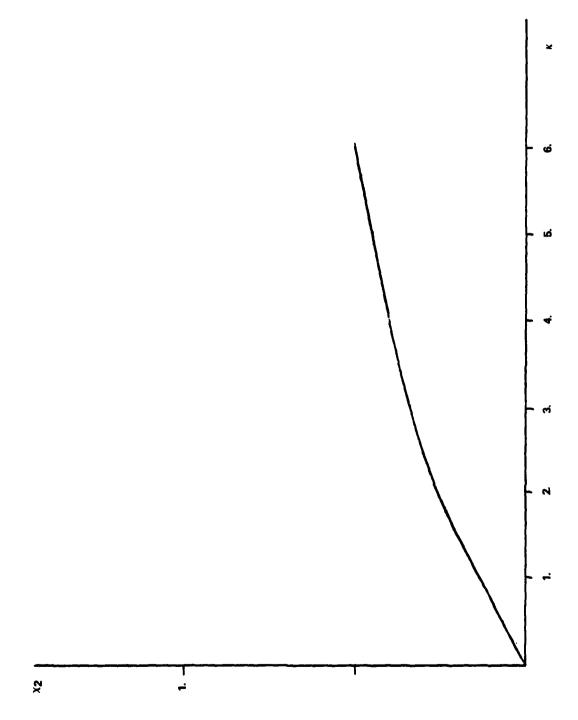


FIGURE 3 EFFECT OF TIME CONSTANT ON THE KINETIC ENERGY AT CUTOFF

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